Frequency, Spectrum and Bandwidth -12.01.2006

Ashutosh Khetan (01CS3013)

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# 1 Line Waves

The general sine wave can be written as

$$s(t) = Asin(2\Pi ft + \Phi)$$

where

- A= amplitude of the wave
- f= frequency of the wave
- $\phi$= phase angle of the wave
- t=time

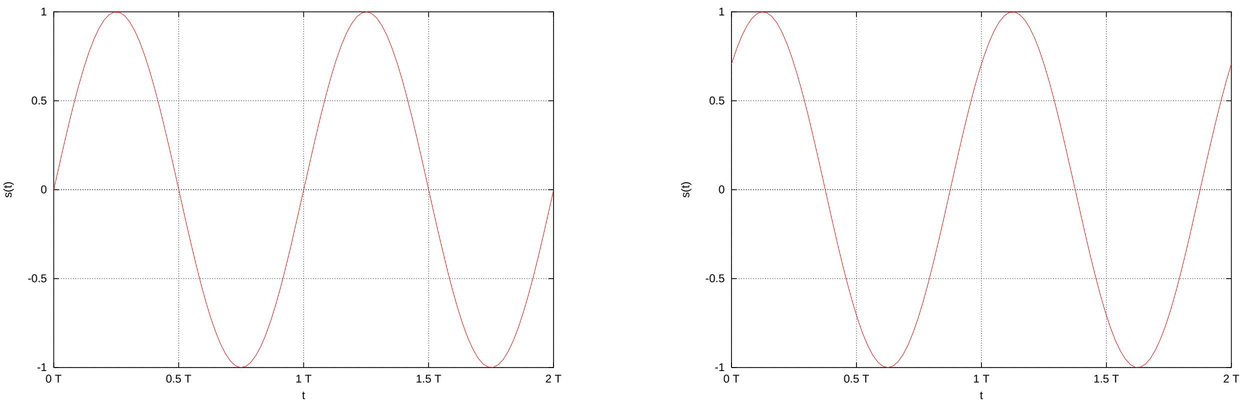


Figure 1: $s(t) = sin(2\pi ft)$ and $s(t) = sin(2\pi ft + \pi/4)$

Infact any periodic wave can be represented as submission of sine waves. What we are really interested in is that how a square wave can be represented by sine waves and what is the frequency range of a square wave.

## 2 Representation of a square wave

We claim that a square wave can be represented as submission of sine waves. Let us justify our claim.

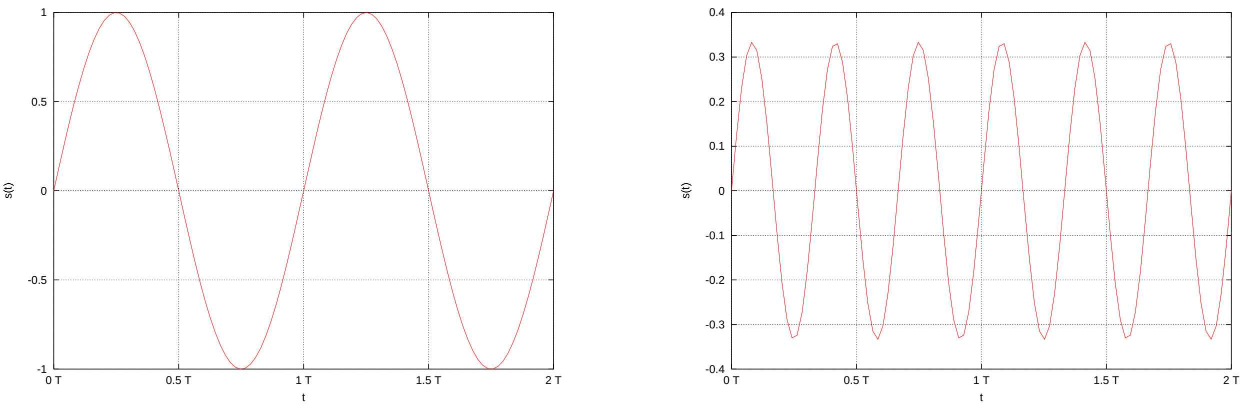


Figure 2: $s(t) = sin(2\pi ft)$ and $s(t) = sin(2\pi(3f)t)$

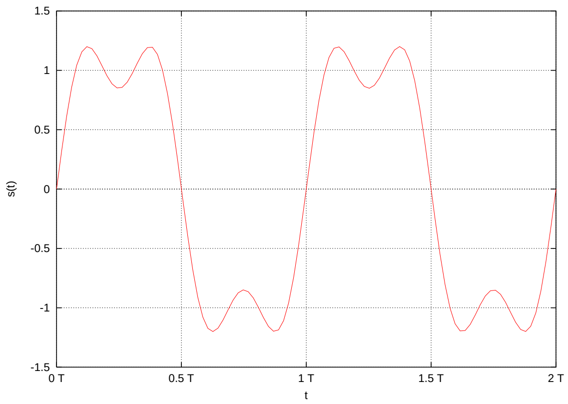


Figure 3: $s(t) = 4/\pi[sin(2\pi ft) + (1/3)sin(2\pi(3f)t)]$

Two important points to be noted are:-

- The second frequency $3f$ is an integral multiple of the first frequency $f$. When all of the frequency components of a signal are integral mutliples of one frequency, the latter frequency is referred to as the **fundamental frequency**.
- The period of the total signal is equal to the period of the fundamental frequency. The period of the component $sin(2\pi ft)$ is $T = 1/f$, and the period of $s(t)$ is also $T$.

We find that $s(t) = 4/\pi[sin(2\pi ft) + (1/3)sin(2\pi(3f)t)]$ is much more closer to a square wave than

$s(t) = sin(2\pi ft)$ or $s(t) = sin(2\pi(3f)t)$. Now lets see what happens for $s(t) = 4/\pi[sin(2\pi ft) + (1/3)sin(2\pi(3f)t) + (1/5)sin(2\pi(5f)t]$ .

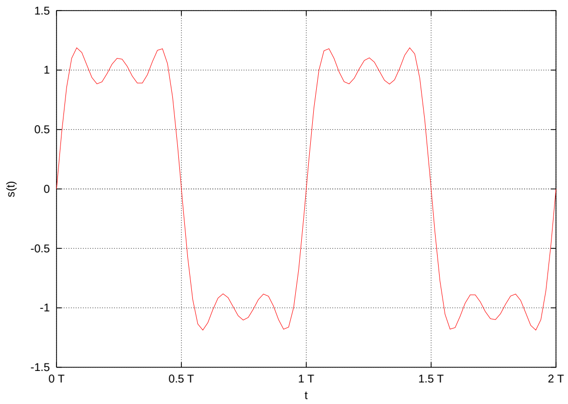


Figure 4: $s(t) = 4/\pi[sin(2\pi ft) + (1/3)sin(2\pi(3f)t) + (1/5)sin(2\pi(5f)t]$

We find that in this case we move more close to square wave. In fact we can represent a square wave with amplitude A and -A as

$$s(t) = A \times \frac{4}{\pi} \times \sum_{kodd,k=1}^{\infty} \frac{sin(2\pi kft)}{k}$$

# 3 Spectrum and Bandwidth

The **spectrum** of a signal is the range of the frequencies it contains. For the signal of $Fig.4$, the spectrum rages from $f$ to $5f$. The **absolute bandwidth** of a signal is the width of the spectrum. For $Fig.4$ the absolute bandwidth is $4f$. Since a square wave can be represented as summation of sine waves of various frequency (as shown above), so it has infinite bandwidth. However most of the energy in the signal is contained in relatively narrow band of frequencies. This band is referred to as **effective bandwidth**, or just **bandwidth**.

# 4 Relationship between Data Rate and Bandwidth

An important issue is that, although a given waveform may contain frequencies over a broad range, as a practical matter any transmission system will be able to accommodate only a limited band of frequencies. This, in turn, limits the data rate that can be carried on the transmission medium.

Let us consider an example. For a square wave with frequency $f$ Hz, two bits are tranferred for every $1/f$ seconds (i.e. per cycle ). Therefore, data rate for a square wave with frequency $f$ is $2f$ bits per second. Now let us see what are the frequency components of this signal. When we use the equation of $Fig.3$, we find that the signal closely resembles a square wave. Here we use two frequencies, $f$ and $3f$. When we use the equation of $Fig.4$, we find that the signal even more closely resembles a square wave. In this case we use three frequencies, $f$ ,$3f$ and $5f$.

As we have seen the equation for a square wave is given by

$$s(t) = A \times \frac{4}{\pi} \times \sum_{kodd,k=1}^{\infty} \frac{sin(2\pi kft)}{k}$$

So a square wave has infinite number of frequency components and thus an infinite bandwidth. However, the peak amplitude of the $kth$ frequency component, $kf$, is only $1/k$, so most of the energy in this waveform is in the first few frequency components.

Now lets take three cases and using it we try to find the relationship between between data rate and bandwidth.

**Case I.**If we represent a square wave by equation of $Fig.4$, then the $bandwidth = 5f - f = 4f$. When $f = 1MHz$, then $bandwidth = 4MHz$ and $data\ \ rate = 2f = 2Mbps$. Thus for a bandwidth of $4MHz$, a data rate of $2Mbps$ is achieved.

**Case II.**If we represent a square wave by equation of $Fig.4$, then the $bandwidth = 5f - f = 4f$. When $f = 2MHz$, then $bandwidth = 8MHz$ and $data\ \ rate = 2f = 4Mbps$. Thus for a bandwidth of $4MHz$, a data rate of $2Mbps$ is achieved. So by doubling the bandwidth, we doubled the potential data rate.

**Case III.**Now we represent a square wave by equation of $Fig.3$,then the $bandwidth = 3f - f = 2f$. When $f = 2MHz$, then $bandwidth = 4MHz$ and $data\ \ rate = 2f = 4Mbps$. Here we have assumed that the difference between a positive and negative pulse in $Fig.3$ is sufficiently distinct and the waverform can be successfully used to represent a sequence of $1s$ and $0s$. Thus, a given bandwidth can support different data rates depending upon the ability of the reciever to discern the difference between $s0$ and $s1$ in the presence of noise and other impairments.

From above we can say that any digital signal has infinite bandwidth. If we attempt to transmit this waveform as a signal over any medium, the transmission will limit the bandwidth that can be transmitted. Also, greater the bandwidth transmitted, the greater the cost. So we arrive at a contradictary situation. Economics and practical reasons dictate that digital inofrmation be approximated by a signal of limited bandwidth., but limiting the bandwidth creates distortions, which makes the task of interpreting the recieved signal more difficult. The more limited the bandwidth, the more the distortion, and the gerater the potential for error by the reciever.

We can generalize that if the data rate of the digital signl is $Wbps$, then a very good representation can be achieved with a bandwidth of $2WHz$. However unless noise is very severe, bit pattern can be recovered with less bandwidth than this.

So there is a direct relationship between data rate and bandwidth. The higher the data rate of the signal, the greater is the required effective bandwidth.Other way round, the greater is the bandwidth of the transmission system, the higher is the data rate that can be transmitted over the system.