Different kinds of Assortative mixing

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1 Application of synthesis algorithm

Synthesis of a random network following a given mixing pattern has many important significance. Assume that we study the airlines of USA and derive the mixing pattern of different USA airports. Now we want to create a new airlines system in Africa which will have similar property as USA airlines. In this case, using the assortative mixing table derived from USA airlines system, we synthesize a new network which connects various airports of Africa. That means we extract the mixing pattern from one environment and apply it to another environment.

2 Other kinds of assortative mixing

Figure 1: Assortative mixing depending on the property of the vertex

Associativity can depend on vertex property other than the simple enumerative properties like race. We define e_{ij} be the fraction of edges in our network that connect a vertex of property x to another vertex with property y. The matrix e_{xy} must satisfy sum rules of the form

$$\sum_{xy} e_{xy} = 1, \sum_{y} e_{xy} = a_x, \sum_{x} e_{xy} = b_y \tag{1}$$

where a_x and b_y are respectively the fraction of edges that start and end at virtices with property x and y. In this case the appropriate defination for the assrtative coefficient is

$$r = \frac{\sum_{xy} xy(e_{xy} - a_x b_y)}{\sigma_a \sigma_b} \tag{2}$$

where σ_a and σ_b are the standard deviations of the distributions a_x and b_y. It can be noted that r is actually Pearson's correlation coefficient for the quantities x and y. It takes values in the range $-1 \leq r \leq 1$ with $r = 1$ indicating perfect assortative mixing and $r = 0$ indicating no correlation between x and y and $r = -1$ indicating perfect disassortative mixing i.e. perfect anti-correlation between x and y.

2.1 Mixing by Vertex Degree

Here we consider the one particular case of mixing that of mixing by vertex degree.Let us define e_{jk} to be the fraction of edges in a network that connect a vertex of degree j to degree k. in fact we define j and k to be the "excess degrees" of the two vertices i.e. the number of edges incident on them less the one edge that we are looking at present. In other words, j and k are one less than the total degrees of the two vertices. If the degree distribution of the network as a whole is p_k then distribution of excess degree of the vertex at the end of the randomly chosen edge is

$$q_k = \frac{(k+1)p_{k+1}}{z} \tag{3}$$

where $z = \sum_k kp_k$ is the mean degree. Then one can define the assortativity coefficient to be

$$r = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma_q^2} \tag{4}$$

where σ_q is the standard deviation of the distribution q_k. On a directed or similar network, where the ends of an edge are not same and $e_j k$ is asymmetric, this generalizes to

$$r = \frac{\sum_{jk} jk(e_{jk} - q_j^a q_k^b)}{\sigma_a \sigma_b} \tag{5}$$

where σ_a and σ_b are standard deviation of the distributions q_k^a and q_k^bb for the two types of ends.

3 Real World Example

Network	type	size n	assortativity r
Physics Co-authorship	undirected	52909	.363
Biology co-authorship	undirected	1520251	.127
mathematics co-authorship	undirected	253339	.120
film actor collaborations	undirected	449913	.208
company directors	undirected	7673	.092
email address books	directed	16881	.092
Internet	undirected	10697	-.189
WWW	directed	269504	-.067
software dependencies	directed	3162	-.016
protein interactions	undirected	2115	-.156
metabolic network	undirected	765	-.240

Above table shows the values of r measured for a variety of different real world networks. The networks shown are mainly social (different co-authorship network,film acot, compant diretors collaboration), technological(Internet, WWW) and biological(protein interaction, metabolic networks) networks. a particular feature of the table is that the values of r for the social networks are all positive, indicating assortative mixing by degree while those for the technological and biological networks are all negative indicating disassortative mixing.

4 References

1. "Mixing patterns and community structure in networks", M. E. J. Newman and M. Girvan, in Statistical Mechanics of Complex Networks, R. Pastor-Satorras, J. Rubi, and A. Diaz-Guilera (eds.), Springer, Berlin (2003).
2."Assortative mixing in networks", M. E. J. Newman, Phys. Rev. Lett. 89, 208701 (2002).