Complex Network Theory
Social Network Theory - Cliques and Clans

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1 Introduction

Cliques are complete graphs. In a social network, it is often found that people mix in a closed group, i.e. everybody in the group knows each other. To represent this idea in the network, we use the concept of clique. But as such the concept of complete graph in a social network is too strict condition to be imposed. So we go for certain relaxation in the concept.

2 Clique

Cliques are complete graphs where all the nodes are connected to each other. But in order to relax the criterion we introduce the concept of *n-clique*. It is defined as follows
An n-clique is the **maximal subset** *of the nodes where distance between any two nodes u and v is less than or equal to n.*

$$d(u, v) \leq n, \forall u, v$$

Suppose in Fig 1 we find that there exists an n-clique (here $n = 2$). The 2-clique is $\{a, b, c, e, f\}$. Note that the $d(e, c) = 2$ through the vertex d. So the 2-clique is depending on some vertex which is outside the set. In order to avoid this, we introduce the concept of n-clan.

3 Clan

We define the n-clan as follows.
An n-clan is an n-clique with diameter(D) less than n.

$$D \leq n$$

In the Fig 1, we find that the set $\{b, c, d, e, f\}$ comprises the 2-clan. This set is *maximal* and everybody can reach other vertices via this closed set only.

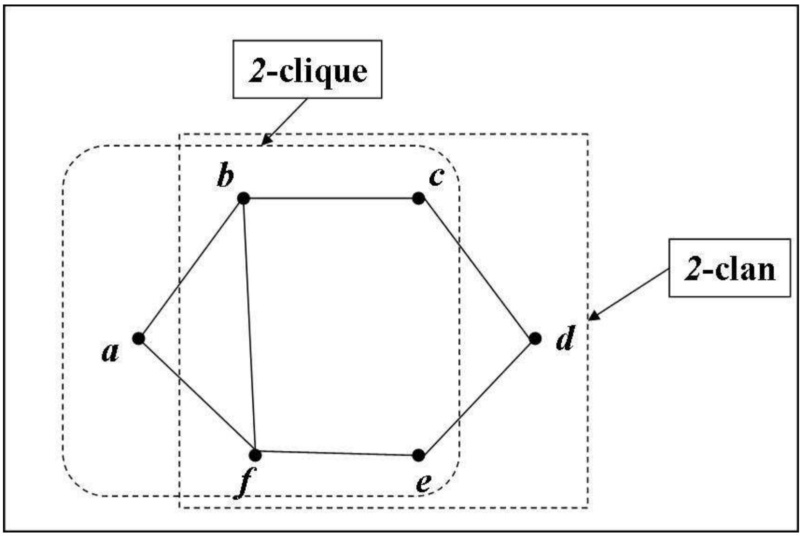

Figure 1: 2-clique and 2-clan

4 K-plex

We define the K-plex as follows.
In a K-plex, all the nodes have degree at least $(n-k)$. So, 1-plex represnts a clique.

Theorem 1: If $k < \frac{n+2}{2}$ then, diameter (D) is not more than 2.

$$k < \frac{n+2}{2} \Rightarrow D \leq 2$$

Proof: Let, $N(u)$ represent the set of adjacent vertices to u. Now from the given conditions we get,

$$d(u) \geq n-k > \frac{n}{2} - 1$$
$$\Rightarrow d(u) \geq \frac{n}{2}$$

For any two vertices u and v,

$$\begin{aligned} N(u) \cup N(v) &= N(u) + N(v) - N(u) \cap N(v) && (1) \\ \Rightarrow N(u) \cap N(v) &\geq \frac{n}{2} + \frac{n}{2} - (N(u) \cup N(v)) && (2) \\ &\geq n - (n-2), \text{ from Pigeon Hole Principle} && (3) \\ &= 2 && (4) \end{aligned}$$

Hence, we get that there exists a common neighbor to any random selection of (u, v) pair. So the diameter cannot be more than 2.

$$D \leq 2 \text{ proved.} \square$$

5 K-core

Lastly, we define K-core. It is defined as follows.
A K-core is defined as the maximal subset where each node is connected to at least K members. The minimum size of the maximal subset is $(K+1)$.

Note that,

$$(n-k) - plex \subset k - core$$