Complex Network Theory Social Network Theory - Community Structure

Lecture delivered by Prof. Niloy Ganguly

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1 COMMUNITY FORMATION

Identifying communities/clusters

- Traditional approaches
- Social science approaches Both the point above means....Avoid comparing algorithms.
- What is the measure of a cluster

2 SPECTRAL GRAPH THEORY

Spectral Bisection method If there distinct communities.

2.1 Lagranges's Matrix(L)

If d_i is degree of node i then Lagrange's matrix is defined as follow:

$$L_{ij} = \begin{cases} d_i & \text{if } i = j; \\ -1 & \text{if } i \text{ is connected to } j; \\ 0 & \text{Otherwise.} \end{cases}$$

We can also write L = D-Awhere D is diagonal matrix and A is Adjacency matrix. For the graph shown in Figure 1 below, L matrix will be

$$L = \begin{pmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 3 \end{pmatrix}$$



Figure 1: Graph for matrix L



Figure 2: Community in Laplacean matrix L

Communities/Distinct components: If there is community of Laplacean matrix L, any row or column sum = 0. Communities in Laplacean matrix L is shown in figure 2.

$$AX = \lambda X$$

where $\lambda = \text{Eigen value}$ and X = Eigen Vector.

• (A) Eigen Values of $L \ge 0$

$$L - A = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$
$$det(A - IX) = \begin{pmatrix} 1 - x & -1 \\ -1 & 1 - x \end{pmatrix}$$
$$= (1 - x)^2 - 1$$
$$= x^2 - 2x$$
$$= x(x - 2)$$

Eigen Value, $\lambda = 0$, 2 If L is having communities, then for $\lambda=0$, we have two Eigen Vector

$$X_1 = \begin{pmatrix} 1\\1\\1\\0\\0 \end{pmatrix}$$
$$X_2 = \begin{pmatrix} 0\\0\\0\\1\\1 \end{pmatrix}$$

But here we have connected graph.

- single component
- So we need
- Weakly coupled components
- (B) Eigen values are close to zero
- (C) Except for the first Eigen vector Eigen vectors are orthogonal form real symmetric matrix.

Find

- 2_{nd} smallest eigen value
- find the corresponding eigen vector
- rows which are +ve will be in one community
- rows which are -ve will be in one community

When λX then nature of λX will remain same.